3D Analysis of Static and Dynamic Scenes with Variational Multigrid Techniques

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Ⅰ. Introduction

3D analysis, reconstructing scene structure and 3D motion, of static and dynamic scenes is an important problem in the computer vision. Variational approaches have been recently introduced to directly reconstruct scene depth and 3D motion field from the spatio-temporal change of image sequences. However, variational methods naturally lead to large scaled and challenging numerical linear algebra problems. We adopt a fast numerical scheme based on the full multigrid algorithm and suggest a new energy functional to reliably solve our problem by the multigrid method.

Ⅱ. Proposed Variational Multigrid Approach

We can define the 3D analysis problem as an energy minimization problem, specifically minimizing a global energy functional $E=M+R$ with respect to 3D motion field **t** and **w**, where M measures the term of conformity to visual data, and R defines regularizing constraints on **t** and **w**. In this paper we suggest the following energy functional:

$$\begin{aligned} E(\mathbf{t},\mathbf{w}) &= \frac{1}{2\sigma_\delta^2}\int_\Omega \left(I_1^\sigma + \mathbf{c}^{\sigma t}\mathbf{t} + \mathbf{d}^{\sigma t}\mathbf{w} - I_0^\sigma\right)^2 d\mathbf{x} \\ &+ \mu\int_\Omega \frac{1}{\left\|\nabla I_0^\sigma\right\|^2 + 2\nu^2}\sum_{i=1}^{3}\left(t_i^2 + w_i^2\right)^2 d\mathbf{x} \qquad (1) \\ &+ \lambda\int_\Omega \sum_{i=1}^{3}\left(\nabla t_i^t T\left(\nabla I_0^\sigma\right)\nabla t_i + \nabla w_i^t T\left(\nabla I_0^\sigma\right)\nabla w_i\right)^2 d\mathbf{x}, \end{aligned}$$

where T is a variant of data-driven diffusion tensor for the edge-preserving regularization. Using the calculus of variations and the finite difference method, we can obtain a linear PDE system from (1). To solve the linear system, Gauss-Seidel relaxation (GSR) was used in the variational methods. In this work we introduce a new numerical scheme based on the full multigrid algorithm for the V-cycle (FMV), illustrated in Figure 1, where the PDE system is recursively solved by using a bi-directional coarse-to-fine error correction scheme with fast convergence rate and linear computational complexity.

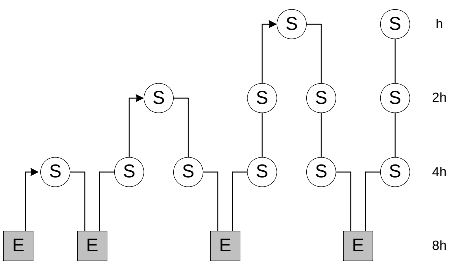

Figure 1. Structure of cycles for the full multigrid algorithm for V-cycle with 4 grids. Each S denotes a smoother and each E denotes an exact solver.

Ⅲ. Experimental Results

The performance of the GSR scheme and our FMV scheme is compared in terms of convergence rate, represented with residual change against iteration number. The result is shown in Figure 2. The estimation results of scene depth of static and dynamic scenes are illustrated in Figure 3.

Ⅳ. Conclusions

A fast and reliable variational multigrid method has been proposed in this paper for directly recovering scene depth and 3-D motion field from the spatio-temporal change of image sequences. In our method depth maps of static and dynamic scenes have been estimated up to scale within a single variational multigrid framework. Future research will include the investigation of a fast and reliable variational multigrid algorithm to directly estimate 3D motion field from image sequences with large displacements.

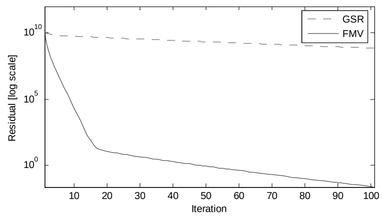

Figure 2. Log-scaled L_2 – norms of residual against iteration number for comparing the convergence rate of the GSR scheme and our FMV scheme.

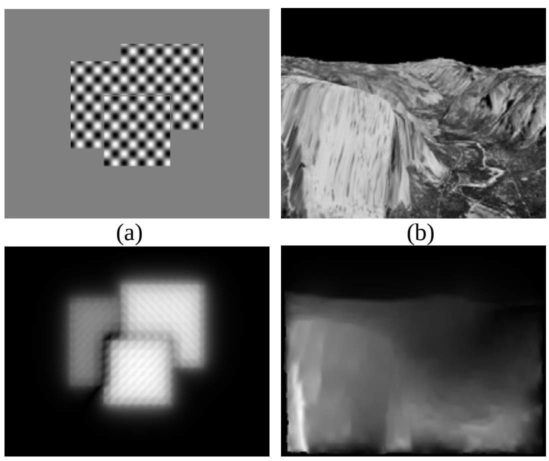

Figure 3. Experimental results (a): *Synthetic* sequence. (b) *Yosemite* sequence. (c), (d): Depth maps after 10 FMV iterations.